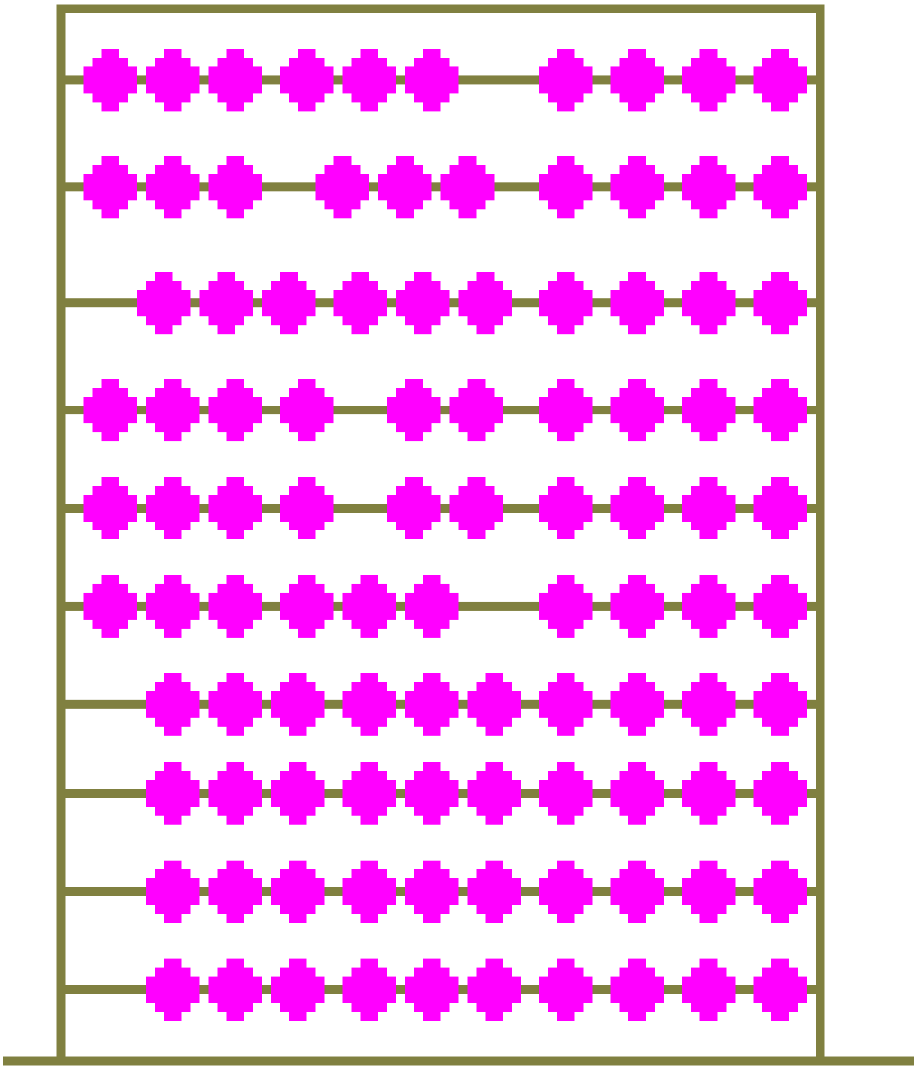
# Mathematics



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Mathematics

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To count one needs numerals. They do not necessarily need to look like the numerals we write: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. There are former, and sometimes still used systems, with only one character: I. Such systems were first used long before there were paper and pencils. People counted by carving lines on objects like a piece of wood. It worked fine when the only thing that needed to be counted was simple stuff like if every sheep was still in the heard. And such systems are still used today by many when listing points in card games, see for example the following three ways to record the score for four rounds of the Swedish card game “Chicago”:

Method 1. The usual counting with our standard numerals:

| Jonas | Anton | Kalle | Johanna | Sara |
|---|---|---|---|---|
| 2+2=4 | 0 | 5 | 0 | 3 |
| +5= 9 | +2+2= 4 | +3= 8 | +0= 0 | +0= 3 |
| +0= 9 | +2= 6 | +3= 11 | +0= 0 | +5= 8 |
| +5= 14 | +0= 6 | +0= 11 | +3+3+3= 9 | +0= 8 |

Method 2. Counting by adding lines after each other in a row:

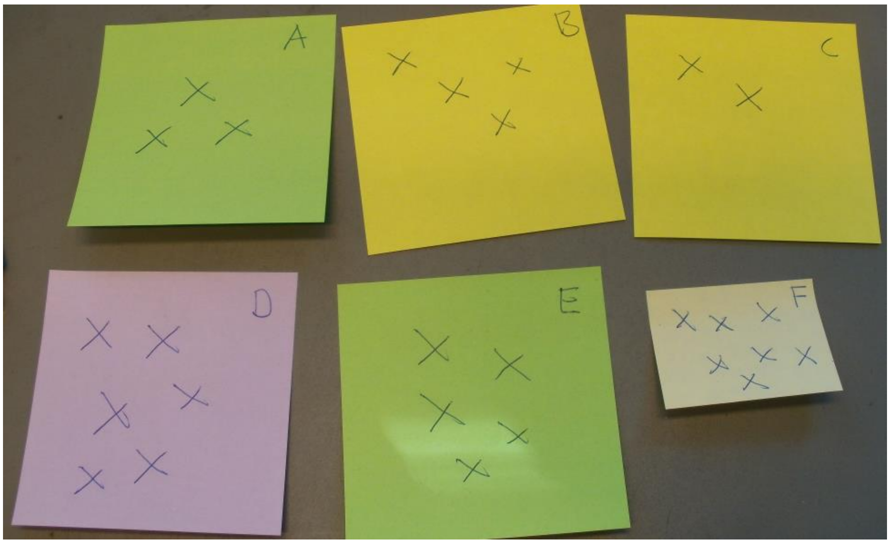
| Jonas | Anton | Kalle | Johanna | Sara |
|---|---|---|---|---|
| IIIIIIIIIIIIII | IIIIII | IIIIIIIIIII | IIIIIIIII | IIIIIIII |

Method 3. Counting by adding lines after another up to four, then an oblique line over the four and then another round with five lines, then a new row:

| Jonas | Anton | Kalle | Johanna | Sara |
|---|---|---|---|---|
| ~~IIII~~ ~~IIII~~ | ~~IIII~~ I | ~~IIII~~ ~~IIII~~ | ~~IIII~~ IIII | ~~IIII~~ III |
| IIII | I | | | |

Which would you choose if you were the one who would keep the scoring?
At least I would have chosen method 3 for it is simpler than method 1 and more foreseeable than method 2. But why is it more foreseeable?

Look, one at a time, on the post-it notes below and judge how many crosses there are on each of them.




With a single glance you can probably see the amount of crosses on note A, B and C. But on the notes D, E and F, you probably have to count them. If you have to count you're like most people, who can only take in up to four characters without counting.

But method 3 would, however, be very difficult to use on, for example, price tags in a store. Probably in order to facilitate the use of larger the number, we created symbols given value greater than one. In these early systems each symbol always had the same value regardless of the how it was placed in relation to other symbols, in contrast to our system, where a numeral at the beginning of a row of numerals is more worth than one at the end. For example, 9 111 SEK is much more than 1 119 SEK.

One of these old systems is, to some extent, alive even today. It was invented by The Romans[1]. When the Roman system is used today it works like this:

| Value | Symbol |
|---|---|
| 1 | I |
| 5 | V |
| 10 | X |
| 50 | L |
| 100 | C |
| 500 | D |
| 1000 | M |

By combining these seven numerals in different ways we get different numbers, like for example:

| Our numerals | Roman numerals | System with only one character |
|---|---|---|
| 15 | XV (= 10+5) | ~~IIII~~ ~~IIII~~ ~~IIII~~ |
| 75 | LXXV (= 50+10+10+5) | ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~<br>~~IIII~~ ~~IIII~~ ~~IIII~~ |
| 178 | CLXXVIII<br>(= 100+50+10+10+5+1+1+1) | ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~<br>~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ |

As long as the numbers were so small that the actual counting operation could be done in the head, the system was probably working decently. But when the numbers are so large that we can not manage to keep them in our heads, it gets worse. The problems decreased sharply when the Indies[2] found out the numerals and the system that we use today.

1. The Romans were those who were citizens of a great and for the time very advanced empire called The Roman Empire that covered all the coasts of the Mediterranean around the year zero of our era.

2. Unfairly these numerals are called Arabic numerals.

Though today we often use numbers that are so large that they become intractable even with these nifty numerals. This problem we solved through introducing various special characters (table 1) in order to avoid having to count the less important figures following the first one.

Table 1. Numeral factor and prefix.

| Numeral factor | As ten potens | Prefix | Symbol |
|---|---|---|---|
| 1 000 000 000 000 | $10^{12}$ | Terra | T |
| 1 000 000 000 | $10^{9}$ | Giga | G |
| 1 000 000 | $10^{6}$ | Mega | M |
| 1 000 | $10^{3}$ | kilo | k |
| 100 | $10^{2}$ | hekto | h |
| 10 | $10^{1}$ | deka | da |
| 0.1 | $10^{-1}$ | deci | d |
| 0.01 | $10^{-2}$ | centi | c |
| 0.001 | $10^{-3}$ | milli | m |
| 0.000001 | $10^{-6}$ | micro | μ |
| 0.000000001 | $10^{-9}$ | nano | n |
| 0.000000000001 | $10^{-12}$ | piko | p |

Although the numbers are large, the vast majority of the mathematical calculations needed for the community and all the appliances to work, still simple. It is almost always enough to master addition, subtraction, multiplication and division.

The large numbers are all around us in our daily lives. And they are often readings from measurements of such things can be measured (called variables), as energy consumption, distance, storage capacity of computers, and the transmission frequencies (known as carrier frequencies) for radio stations.

Energy consumption in one year in a house = 26 321 000 watt-hours (abbreviated Wh) = 26 321 kWh.

Distance Stockholm-Uppsala = 71 000 meter (abbreviated m) = 71 km.

The storage capacity on a hard drive = 3 000 000 000 000 bytes (abbreviated b) = 3 Tb.

The transfer frequency for P1 in Stockholm = 92 500 000 hertz (abbreviated Hz) = 92.5 MHz.

Wh, m, b, Hz are units that we have agreed upon, since it is easier and more accurate to say 60 watt-hours than, for example: The electrical energy needed to power a 60 W's light bulb for one hour.

To describe how different variables affect each other we have decided to use technical formulas. A commonly used one is the one we use to calculate the travel time between two places (the sought greatness in this case, time): The time it takes to travel by car between two places = the distance between them divided with the average speed on the trip, i.e.:

$$\text{The time (in hours)} = \frac{\text{distance (in km)}}{\text{speed (in km/h)}}$$

It becomes clearer when it is written with abbreviations and acronyms commonly used in this case:

$$t = \frac{s}{v}$$

The formula shows that the longer the distance, or the lower the average speed is, the longer it will take to arrive.

And so it is with formulas in general. They describe how things relate to each other. In table 2 a few more or less useful formulas are shown.

Table 2. Some formulas with description of what it means, and examples of metrics.

| Variable | Formula | Which means | Examples of measured values |
|---|---|---|---|
| Acceleration (a) | a=dv/dt, where dv= the speed increase, dt= the time for the speed increase | The shorter the time it takes to change the speed, the greater is the acceleration (or the opposite deceleration, thus braking). | 5,5 $m/s^2$ (= 0 - 100 km/h on 5 s) |
| Density (ρ) | ρ=m/V, m= weight, V= volume | A little thing has grater density than a bigger thing weighing the same. | 1 $kg/dm^3$ |
| Fluid pressure (p) | p= ρ gh, ρ= fluid density, g= gravity (= 9,81 $m/s^2$, h= fluid height. | The higher the fluid is above something or the heavier the fluid is the higher is the pressure on it. | 100 kPa (=100 000 $N/m^2$= 1 bar) |
| Mechanical pressure (p) | p=mg/A, m= weight of the load, g= gravity, A= the loaded area | The heavier the load or the smaller the area the higher pressure on the area. | 100 kPa (=100 000 $N/m^2$= 1 bar) |

In addition to numbers, parameters, units of measurement and formulas we also need materials to create something more than just thoughts. The most common building materials in our world are carbon, hydrogen and oxygen that are mixed in different ways. These three substances are called elements because they can not be separate into other substances.
But they are still mixtures of atomic particles. Hydrogen is the simplest element. In its basic form it has only two atomic particles: a proton and an electron. The proton and the electron are held together into a hydrogen atom by having different charge, as the poles of a magnet. If the proton is like a small ball in the middle of a giant football stadium, the electron is even smaller and it spins in the top row of the bleachers. Most of the atom is thus void.
The hydrogen atom and all other elements may also have an additional type of particles called neutrons. Hydrogen atoms with neutrons are called isotopes of hydrogen and there are those who have one neutron (deuterium) or two neutrons (tritium).
The next simplest element (helium) has two protons and electrons. The additional next is called lithium and in total there are about 100 elements, of which most are metals. To describe them, we invented the "periodic table". A simple version it looks like this:

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | | | | 2 |
| 3 | 4 | | | | | | | | | | | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | | | | | | | | | | | 13 | 14 | 15 | 16 | 17 | 18 |
| 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 |
| 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 |
| 55 | 56 | * | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 |
| 87 | 88 | ** | | | | | | | | | | | | | | | |

* No. 57-71 called lanthanides and they are presented separately partly because they are very rare and partly because leaving them out is considered to make the system easier to grip.

** No. 89-103 called actinides are so rare metals that only four of them at all occur in nature (No. 89-93), the rest can only be produced by nuclear reactions.

The number in each box represents the number of protons found in the nucleus of each atom of the element and thus the number of electrons in its ground state (see below). Instead of the number of protons, one can present the periodic table with the elements names:

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hydro-gene | | | | | | | | | | | | | | | | | He-lium |
| Litsium | Beryl-lium | | | | | | | | | | | Lives | Carbon | Nitro-gen | Oxy-gene | Fluo-rine | Neon |
| Sodium | Magnesi um | | | | | | | | | | | Alu-minium | Silicon | Phosp-horus | Sulfur | Chlo-rine | Argon |
| Potass-ium | Calcium | Skan-dium | Titan-ium | Vana-dium | Chro-mium | Mangan ese | Iron | Cobalt | Nickel | Copper | Zinc | Gallium | Germ-anium | Arsenic | Sele-nium | Bro-mine | Kryp-ton |
| Rubi-dium | Strontiu m | Yttrium | Zirco-nium | Niob-ium | Molybd enum | Tek-netium | Rute-nium | Rhodiu m | Palla-dium | Silver | Cad-mium | Indium | Tin | Anti-mony | Tell-urium | Iodine | Zenon |
| Cesi-um | Barium | * | Haf-nium | Tan-talum | Tung-sten | Rhe-nium | Os-mium | Irdium | Platinu m | Gold | Mer-cury | Tallium | Lead | Bis-muth | Polo-nium | Asta-tine | Radon |
| Franc-ium | Radium | ** | | | | | | | | | | | | | | | |

Some elements resemble each other more than others. For example, gold has more in common with silver than with aluminium. Therefore, they are divided into different ways, here an example of how it can be done:

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hydrogen | | | | | | | | | | | | | | | | | Helium |
| Lithium | Beryllium | | | | | | | | | | | Lives | Carbon | Nitrogen | Oxygen | Fluorine | Neon |
| Sodium | Magnesium | | | | | | | | | | | Aluminium | Silicon | Phosphorus | Sulphur | Chlorine | Argon |
| Potassium | Calcium | Scandium | Titanium | Vanadium | Chromium | Manganese | Iron | Cobalt | Nickel | Copper | Zinc | Gallium | Germanium | Arsenic | Selenium | Bromine | Kryp-ton |
| Rubid-ium | Strontium | Yttrium | Zirco-nium | Niobium | Molybdenum | Technetium | Ruthenium | Rhodium | Palla-dium | Silver | Cad-mium | Indium | Tin | Antimony | Tellurium | Iodine | Xenon |
| Caesium | Barium | * | Haf-nium | Tantalum | Tungsten | Rhenium | Os-mium | Iridium | Platinum | Gold | Mercury | Thallium | Lead | Bismuth | Polo-nium | Astatine | Radon |
| Franci-um | Radium | ** | | | | | | | | | | | | | | | |

- Non metallic materials
- Noble gases (they resemble in that they normally occur in the form of gas and that they hardly react with other elements)
- Noble metals (they hardly react with other elements and many of them have high density).
- Light metals (they are lighter than other metals)
- Other metals, of which the rare:

Those who mix different elements to mixtures like water, plastic, gasoline and so on, uses various methods and, of course, recipes for what to mix. These recipes are not:

Take two hydrogen atoms.
Take an oxygen atom.
Pour them into a bowl and stir to get a water molecule.

Since atoms are so small that even with the most advanced instruments we can't take only one atom. In addition, recipes in chemistry are not written that way, one reasons is the chemists use the elements abbreviation.

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H | | | | | | | | | | | | | | | | | He |
| Li | Be | | | | | | | | | | | B | C | N | O | F | Ne |
| Na | Mg | | | | | | | | | | | Al | Si | P | S | Cl | Ar |
| K | Ca | Sc | Ti | V | Cr | Mn | Fe | Co | Ni | Cu | Zn | Ga | Ge | As | Se | Br | Kr |
| Rb | Sr | T | Zr | Nb | Mo | Tc | Ru | Rh | Pd | Ag | Cd | In | Sn | Sb | Te | I | Xe |
| Cs | Ba | * | Hf | Ta | W | Re | Os | Ir | Pt | Au | Hg | Tl | Pb | Bi | Po | At | Rn |
| Fr | Ra | ** | | | | | | | | | | | | | | | |

So the recipe for a water molecule could look like this:

$H + H + O = H_2O$

Different elements react with each other because some want to let go of electrons while others like to get more. The electrons spin around at various, more or less fixed distances from the protons in the nucleus. Next to the core rotates, at a maximum, two electrons. On the next orbit distance spins up to 8 and on the further next up to 10 electrons. In the periodic table this is described so that each row in the system will be a new orbital distance (it has partially been thumbed on to make the system transparent). The element at the left of each row has only one electron in its outermost orbital distance, while the element along the right has the maximum number of electrons in it. The model fits well with the fact that the leftmost substances very easily reacts with other substances, while the ones to the far right does not react at all.

The elements at the left reacts by relinquish the lone electron in the outer orbital distance. The elements next to the right instead like to take up an electron so that their outer orbital distance becomes full. Atoms that have given up or received extra electrons are called ions.

The number of electrones per atom, of the element to the very right.

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | H | | | | | | | | | | | | | | | | | He |
| 10 | Li | Be | | | | | | | | | | | B | C | N | O | F | Ne |
| 18 | Na | Mg | | | | | | | | | | | Al | Si | P | S | Cl | Ar |
| 36 | K | Ca | Sc | Ti | V | Cr | Mn | Fe | Co | Ni | Cu | Zn | Ga | Ge | As | Se | Br | Kr |
| 54 | Rb | Sr | T | Zr | Nb | Mo | Tc | Ru | Rh | Pd | Ag | Cd | In | Sn | Sb | Te | I | Xe |
| 86 | Cs | Ba | * | Hf | Ta | W | Re | Os | Ir | Pt | Au | Hg | Tl | Pb | Bi | Po | At | Rn |
| 118 | Fr | Ra | ** | | | | | | | | | | | | | | | |

Emits electrons in the outermost orbit very easily
Emits electrons in the outermost orbit pretty easily
Receive electrons in the outermost orbit very easily
Receive electrons in the outermost orbit pretty easily
Neither emits or receives electrons

This phenomenon we use to create blends between different elements. We do this by attracting some elements to give up electrons that other elements are attracted to receive. Water, for example, is made in some fuel cells as a by-product in the generation of electricity. The fuel cell basically consists of a container filled with an electrically conductive liquid. In both ends of the container is an electrode which is connected in the same way as a conventional battery, to the plus and minus terminals of the thing to be driven. Adjacent to the electrode that is coupled to the positive terminal hydrogen gas is injected and adjacent to the other electrode oxygen gas or air is injected. Since hydrogen gladly relinquishes its electron the hydrogen splits into hydrogen ions and electrons. Since oxygen gladly takes up two extra electrons, the electrons move towards the oxygen through the pipe and the thing to be driven thereby forming an electrical current.

The hydrogen ions have the same charge as a lone proton, while the oxygen atom that received two electrons (oxygen ion) has the opposite charge. The oxygen ions and hydrogen ions then move towards each other in the conductive fluid and connect into water ($H_2O$).

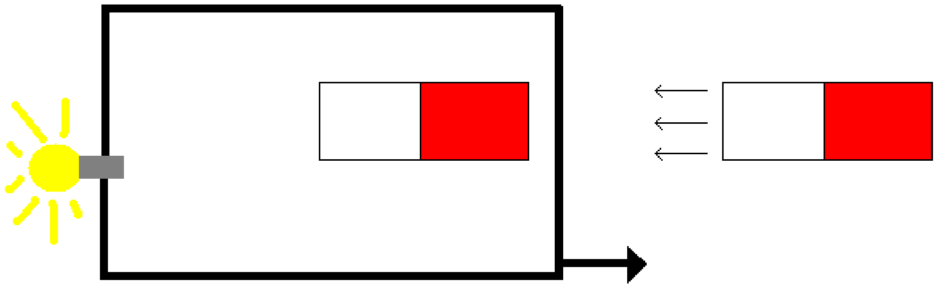
To make a certain amount of water, it takes exactly twice as many units of hydrogen as oxygen atoms. But it is not, as previously mentioned, possible to pick them atom by atom and put them together. We have therefore introduced the greatness amount of substance, which is measured in moles. 1 mole contains $6{,}022 \times 10^{23}$ units. How many atoms as a unit consisting of vary depending on how the atoms bind together.

Both hydrogen atoms and oxygen atoms prefer to bind together in pairs, since the pairs can share the electrons needed to fill the outermost orbit.

A unit hydrogen therefore looks like this: $H_2$ and oxygen: $O_2$. One mole of $H_2$ weighs 2 grams (g) while a mole $O_2$ is weighing 32 g, the recipe for the water thus becomes:

| | $2H_2$ | + | $O_2$ | $\longrightarrow$ | $2H_2O$ |
|---|---|---|---|---|---|
| The number of atoms/ unit (pieces) | 2 | | 2 | | 3 |
| The number of atoms/mole | $12{,}044 \times 10^{23}$ | | $12{,}044 \times 10^{23}$ | | $18{,}066 \times 10^{23}$ |
| Amount of substance (mole) | 2 mole | | 1 mole | | 2 mole |
| Substance weight (g) | 4 g | | 32 g | | 36 g |

Electrons can also be brought to move using magnetic fields. Since they are negatively charged they will want to move to the positive side of the field. And it is this desire that allows us to create electricity in generators. The usable electricity is created through that "ring" made of an electrically conductive material is moved against the movement direction of the electrons in the magnetic field. Then start the electrons inside the cable to move through the ring, which have the effect that it is electrified. In the ring there is something that we want to electrify. At the same time a magnetic field is created around the conductor and this field becomes opposite to the first field and it makes it harder for them to move. The ring is called conductor and the things we want to electrify are called loads.



The stronger the magnetic field, the faster the electrons are moving in the conductor. It is describes by saying that the voltage increases. The greatness voltage is abbreviated V and is measured in volts (symbol V).

The more electrons moving in the conductor, the stronger is the current in it. The greatness current is abbreviated I and it is measured in amperes (symbol A). The current in its turn also depends on how strong the magnetic field is, but also how easily electrons can move in the conductor. Electrons in a conductor (wire) namely constantly meet obstacles (other atomic particles in conductor) and thus a kind of friction occurs. This friction creates heat. The friction is called resistance (abbreviated R) and the unit is Ohm. In insulators, the resistance is very big while it is small in good leaders, such as copper wire. The relationship between voltage, resistance and current can be described by the formula:

$$I=\frac{U}{R}$$

The formula, called Ohm's law after the man who came up with it, can, of course, also is written:

$$U = I \times R$$

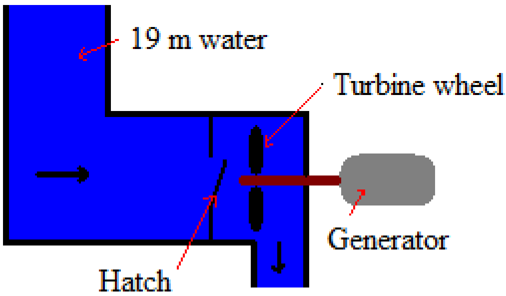
But the power that can be obtained in the conductor is, as mentioned, not only dependent on the voltage and the resistance in the conductor, but also the power of the magnetic field that the conductor moves in. The power is called effect (written P) and the unit is watts (Symbol W). The formula for the currents relation to the effect and voltage is:

$$I=\frac{P}{U}$$

Power generators are based on the fact that when an electrically conductive material (a conductor) is moving in a magnetic field it creates current in the conductive material. In small generators the magnetic field is created by permanent magnets, but for example in many hydro power plants, the magnetic fields created by electricity. The generators of these look like large electric motors, with a rotating part in the middle (the rotor). The rotor is provided with conductors through which a small current is led, and thus a magnetic field is created around the rotor. Around the rotor thick conductors are mounted. As the rotor rotates, the magnetic field around it will rotate to, which forces the electrons in the thick conductors around the rotor to move too, and there is current in them.

The effect is most the important factor in the electrical system, because one can not take out more power than what is created. The voltage and current, however can be changed with transformers.
The total resistance of the system constitute of the sum of resistances in the devices connected to the system and in the system itself. The resistance of the system should be as low as possible to avoid wasting power, while the connected devices, often should have as high resistance as possible to hold down the power through them.
When Sweden was electrified in the early 1900s villages around the country built their own electric networks, which was supplied from small hydroelectric power plants nearby.

The effect that could be taken out of the hydro power plant was due to the water pressure on the turbine wheel and the amount of water that flowed in the river, and how little energy that disappeared on its way from the turbine wheel to the power cables from the generator (which is expressed as the total efficiency). The water pressure depends in its turn on how much height difference there is between the water surface in the pond and the turbine wheel. And efficiency is depending on which components the facility is built with.




Effect = fall height (h) x water flow (Q) x acceleration of gravity (g) x power plant efficiency (n)

With the current symbol, the formula is as follows:

$P = H \times Q \times g\ (=9.81\ m/s^2) \times n$ [kW]

If there were 19 meters of water above the location of the turbine wheel (fall height H = 19 m) and there in average was water enough to let seven hundred litres per second flow through the turbine (the flow rate Q = 0.7 m3/s) and the total efficiency of the equipment the village would purchase was approximately 85%, the effect that the could get would be:

$P_{average} = 19 \times 0.7 \times 9.81 \times 0.85 = 95\ 000\ W = 95\ kW$

Let's say they built one just as large plant near the waterfall. And they used a three phase generator that gave the same power as a conventional three-phase motor was driven with at that time (220 V).
In a three phase generator, the same amount of effect is generated in all three phases and therefore just as much current. The current in each phase would then on average be (if we for the sake of simplicity assume that cos-phi = 1):

$$I_{phase} = \frac{P_{average}}{3 \times U_{phase}} = \frac{95\ 000\ W}{3 \times 220\ V} = 144\ A$$

On the way to the village it would, however, disappear some effect due to its resistance in the conductors. The resistance of a conductor may be 0.00001 Ω/m (10 μΩ/m). If it was, say, 10 km to the village which uses the power, the total resistance in each of the three conductors would then be:

$R_{per\ conductor} = 10\ 000\ m \times 0.00001\ Ohm/m = 1\ Ohm$

The power lost as heat in the conductors would then be:

$P_{loss\ in\ the\ wires}$= 3 x U x I = 3 x R x I x I, since something x something is depreciated something$^2$ the formula becomes:

$= 3 \times R_{per\ wire} \times I^2_{phase} = 3 \times 1 \times 144^2 = 3 \times 20\,736\ W = 62\ kW$

It's not so good, so the villagers raised voltage on the wires to the village with a transformer so that it was 10 000 V in each phase. For thus the current through the wires would be as much less as the voltage was higher. Which would be good because the losses in the line are proportional to $I^2$.

In transformers the transmitted power is the same on both sides (apart from minor losses in the transformer) but the voltage is changed and then current changes too, but in the opposite direction:

$P_{in} = U_{phase1} \times I_{phase1} = P_{out} = U_{phase2} \times I_{phase2}$

The current on the output side $I_{phase2}$ gets per phase:

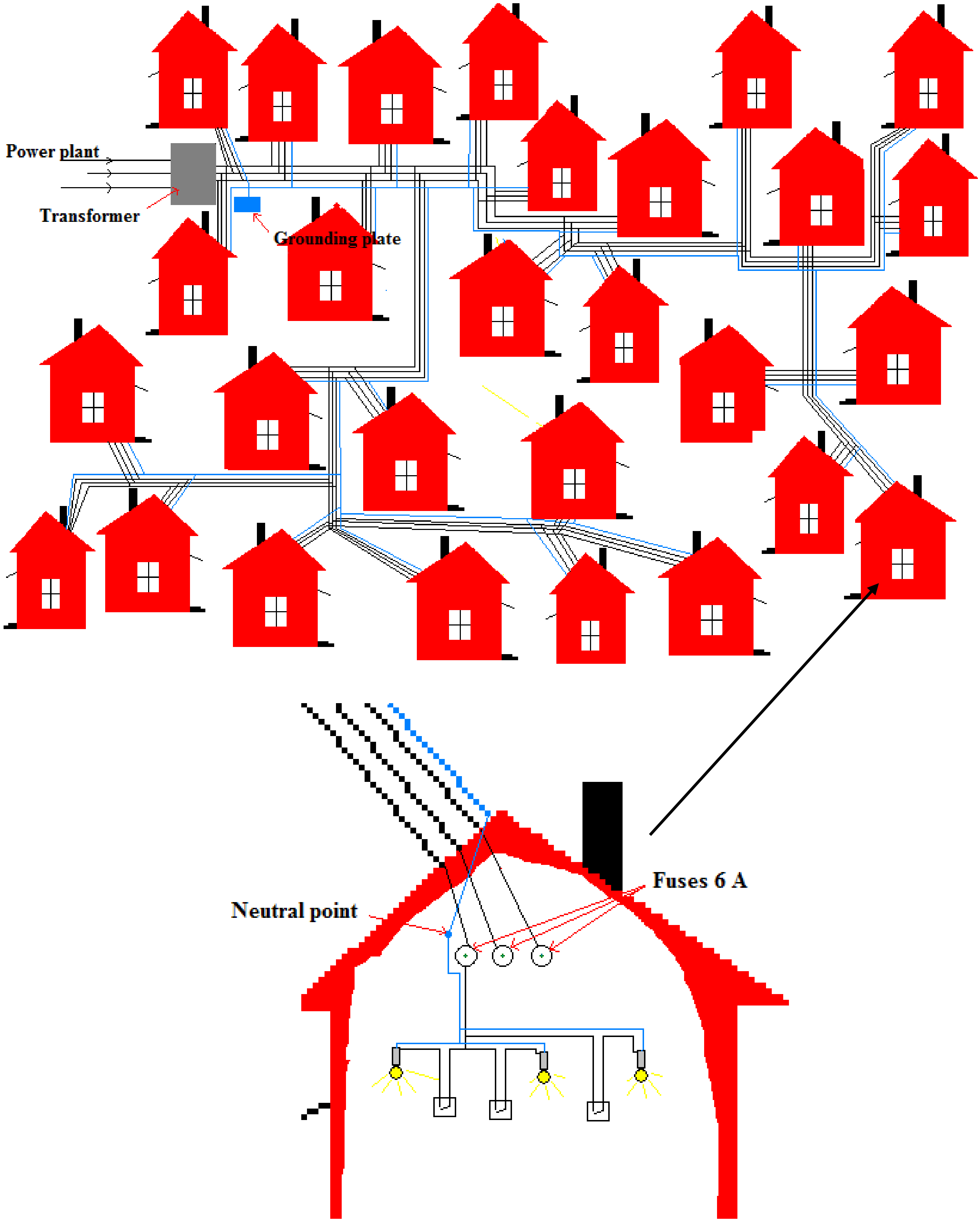
$$I_{phase2} = \frac{I_{phase1} \times U_{phase1}}{U_{phase2}} = \frac{144 \times 220}{10\,000} = 3.2\ A$$

And so they got down losses in the lines to:

$P_{loss\ in\ the\ wires} = 3 \times R_{per\ wire} \times I_{phase}{}^2 = 3 \times 1 \times 3.2^2 = 31\ W$

When the energy is transported to the village it is transformed back to 220 V again, because it was what the villages electric devices were made for, and because it is less dangerous with 220 V than 10 000 V.

At the time when villages had their own power plants, the electric devises they had were mostly light bulbs.



Each house had a fuse of 6 A on each of the three phases. Then they could get:

$P_{total\ in\ each\ house}$= 3 x $I_{fuse}$ x $U_{phase}$= 3 x 6 x 220= 3 920 W= 3.9 kW.

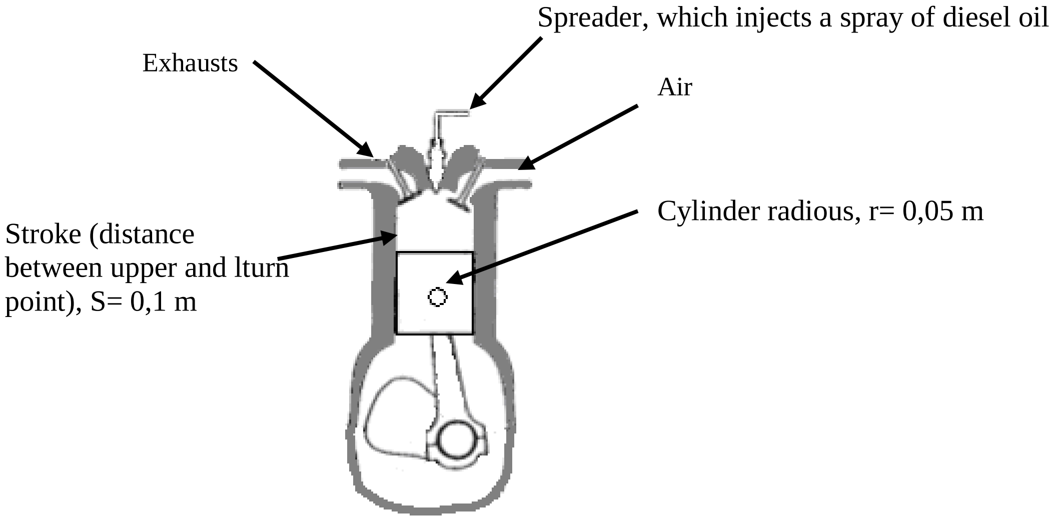
If only they only used the electricity for 60 W light bulbs, they could have over 60 bulbs active at the same time, since:

$$\text{The number of bulbs} = \frac{\text{total effect}}{\text{effect/light bulb}} = \frac{3\,920\ \text{W}}{60\ \text{W/bulb}} = 66$$

Which at the time was a good margin, because they did not have more than a couple of lights bulbs in the ceilings and perhaps a floor lamp. If they had electric cookers, however, it would have been worse. Since they consume, like, 8 kW if all the heating elements are active at the same time.

The less power consumption (i.e. the fewer consumers connected) the higher the resistance (even if a bulb gives high resistance produces a light bulb turned off even higher resistance) and the less total current in the system. This in turn led to that the magnetic field around the thick conductors around the rotor was reduced. Since the field obstructed the field created by the rotor, the rotor then met less resistance and therefore it could spin faster. To counter this, there was always a person sitting in the power station prepared to reduce the flow of water until the rotating part again was spinning at the same speed as intended. So that the generated power was equal to what was consumed.

Other times, they needed instead to take out more power than what could be produced. Therefore, they bought a four-cylinder diesel engine, which they mounted in the power station together with an extra generator.




Say that the engine had a operating speed of 20 revolutions/second, n = 20 revolutions/second

A diesel engine works, as well known, so that the piston is pushed down by an explosion. When the piston goes up again, while the exhaust valve opens and the piston pushes the exhaust gases through it. The piston has so much momentum that it reverses again and continues down. On the way down opens a valve that is sucked in by the descending piston. As the piston rises again the air valve closes and the air is compressed. The more the air is compressed, the hotter it becomes and when the piston is almost up, a spray of diesel oil is injected into the chamber, which is ignited by the hot air. Which in its turn results in an explosion.

The pressure (abbreviated by p) in the cylinder after the explosion is calculated using the formula:

$$p = \frac{n \times R \times T}{V}$$

As a way to write what is called “the ideal gas law" p x V = n x R x T where
p = gas pressure (in $N/m^2$)
V = gas volume (in $m^3$)
n = amount of substance (in moles)
R = gas constant (= 8.3145 J/mole Kelvin)
T = absolute temperature (in Kelvin).

The force on the piston = pressure in cylinder x piston area = p x π x radius x radius = $p\pi r^2$.
The pressure is the largest just after the explosion, and then it decrease as the piston moves downwards. The easiest way, if you want to calculate the generated kinetic energy, is to use the average pressure in the cylinder ($p_{average}$). It is of course possible to compute the mean pressure, though it is much easier to measure the pressure for a few turns and let the measurement computer calculate the mean value.
Let's say that the mean, $p_{mean} = 10 \text{ bar} = 10 \times 10^5 \text{ N/m}^2$.
The effect on the motor axle= the force on the piston x the peripheral speed of the motor axle, and the latter is the same as the stroke (S) x rotation speed (n). Overall, the formula for the effect is:

$P = p_{average} \times pi \times r^2 \times S \times n \times$ the number of cylinders

And this four-cylinder engine with:
The average pressure, $p_{average} = 10 \times 105 \text{ N/m}^2$
the cylinder radius, r = 0.05 m;
stroke length S = 0.1 m;
n = speed of 20 revolutions/second.

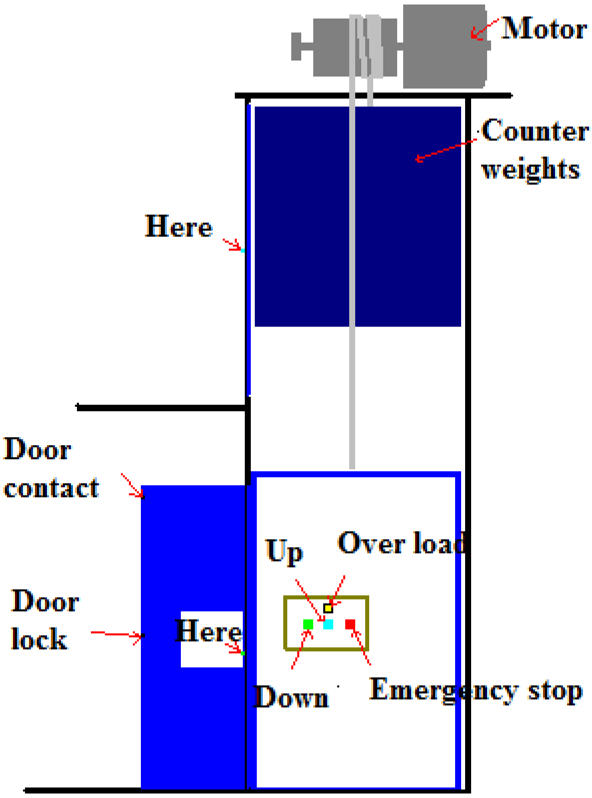
Thus gives:

$P = 10 \times 10^5 \times pi \times 0.05^2 \times 0.1 \times 20 \times 4$ W = 62 800W= 62 kW if the efficiency in the transmission of electrical energy was roughly the same as for the village's hydroelectric system they would get out:

$P_{elektric\ effect} = P_{mechanical\ effect} \times 0.85 = 52.7$ kW

Electronics such as computers and control systems are based largely on an even simpler type of maths that uses only two digits. Say that one of the villagers wanted to build an elevator between two floors and would like it to work like this:

1. The car should move if one of the buttons up or down is pressed.
2. It may only move if the doors are closed.
3. If the emergency stop button is pressed, it shall stop.
4. If there are too many people in the elevator, it shall not start.
5. If the car is heading in one direction, it should not respond if the button pressed for the other direction.
6. If the car is between floors, the doors shall not be possible to open.
7. If the car is on the wrong floor and the "here" button is pressed, the car shall move (if the doors are shut and there are not too many people in it and it is not already on the way).




Then he or she has to get:

An elevator motor.
A car.
Counterweights.
A roll of wire.
Elevator doors.
A contact on each door that is activated when the door is closed.
Electric door locks.
A wave that closes a switch if there are too many people in the car.
Two "here" buttons.
An "up" button in the lift.
A "down" button in the lift.
An "emergency stop" button.
A indicator lamp that is activated when the elevator is too heavily loaded.

For the control of the elevator, a control system is required. Next step on the road to forming such is to find out abbreviations for the various constituent phenomena:

Elevator motor up = MU.
Elevator motor down = MN.
Button up in the car = HU.
Button down in the car = HN.
Door switch up = DU.
Door switch down = DN.
Electric door lock up = EU.
Electric door lock down = EN.
Emergency = N.
Wave to detect overload = Ö.
Lamp that lights up when the car is overloaded = LÖ.
Location contact up = LU.
Location contact down = LN.
Here button up = VU.
Here button down = VN.

The majority of these phenomena generates inputs to the control system: HU, HN, VN, DU, DN, N, Ö, LU, LN, VU, VN.

The elevator builder wanted to make the control system as easy as possible, so he made all the buttons "hold buttons", which means that when they are released the activation stops. Since then he just need five outputs: MU, MN, EU, EN, LÖ.

Each input and output has only two states "On" and "Off" and this can be put into a table that describes what will happen (table 3).

Table 3. State of input and output signals of the different situations that can arise (- means that the input signal can change value without affecting the actual event).

| Row | | In-signals | | | | | | | | | | Out-signals | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | HU | HN | DU | DN | N | Ö | LU | LN | VU | VN | MU | MN | EU | EN | LÖ |
| 1 | Here button pressed upper level | - | Off | On | On | Off | Off | Off | - | On | - | On | Off | On | On | - |
| 2 | The car is on the upper level | - | - | - | - | - | - | On | Off | - | - | - | - | Off | On | - |
| 3 | Someone wants to go down | - | On | On | On | Off | Off | - | Off | - | - | Off | On | Off | On | - |
| 4 | It is too heavy | - | - | - | - | - | On | - | - | - | - | Off | Off | - | - | On |
| 5 | Emergency stop is pushed | - | - | - | - | On | - | - | - | - | - | Off | Off | - | - | - |
| 6 | Here button is pressed lower level | Off | - | Off | On | Off | Off | - | Off | - | On | Off | På | On | On | - |
| 7 | The car is on the lower level | - | - | - | - | - | - | Off | On | - | - | - | - | On | Off | - |
| 8 | Someone wants to go up | On | - | On | On | Off | Off | Off | - | - | - | On | Off | On | On | - |

Instead of "On," we can write "1" instead of "Off" we can write "0" (table 4).

Table 4. On replaced with 1 and Off replaced by 0.

| Row | In-signals | | | | | | | | | | Out-signals | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | HU | HN | DU | DN | N | Ö | LU | LN | VU | VN | MU | MN | EU | EN | LÖ |
| 1 Here button pressed upper level | - | 0 | 1 | 1 | 0 | 0 | 0 | - | 1 | - | 1 | 0 | 1 | 1 | - |
| 2 The car is on the upper level | - | - | - | - | - | - | 1 | 0 | - | - | - | - | 0 | 1 | - |
| 3 Someone wants to go down | - | 1 | 1 | 1 | 0 | 0 | - | 0 | - | - | 0 | 1 | 1 | 1 | - |
| 4 It is too heavy | - | - | - | - | - | 1 | - | - | - | - | 0 | 0 | - | - | 1 |
| 5 Emergency stop is pushed | - | - | - | - | 1 | - | - | - | - | - | 0 | 0 | - | - | - |
| 6 Here button is pressed lower level | 0 | - | 1 | 1 | 0 | 0 | - | 0 | - | 1 | 0 | 1 | 1 | 1 | - |
| 7 The car is on the lower level | - | - | - | - | - | - | 0 | 1 | - | - | - | - | 1 | 0 | - |
| 8 Someone wants to go up | 1 | - | 1 | 1 | 0 | 0 | 0 | - | - | - | 1 | 0 | 1 | 1 | - |

There are two rows that describes the situations when the elevator motor must rotate so that the elevator goes up: row 1 and row 8. Line 1 describes what shall happen if the here button on the upper floor (VU) is pressed (VU = 1), and row 8 describes that to happen if the up button in the car (HU) is pushed (HU = 1). In both cases, the emergency stop (N), overload wave (Ö) and location contact up (LU) shall be = 0, door contacts (DU and DN) must, however, be = 1. Other inputs can change state without affecting the car ride.

Table 5. Situations in which the motor will rotate so that the elevator goes up.

| Row | In-signals | | | | | | | | | | Out-signals | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | HU | HN | DU | DN | N | Ö | LU | LN | VU | VN | MU | MN | EU | EN | LÖ |
| 1 Button pressed upper level | - | 0 | 1 | 1 | 0 | 0 | 0 | - | 1 | - | 1 | 0 | 1 | 1 | - |
| 8 Someone wants to go up | 1 | - | 1 | 1 | 0 | 0 | 0 | - | - | - | 1 | 0 | 1 | 1 | - |

So in row 1, when HN = N = E = LU = 0 and DU = DN = VU = 1, the engine shall rotate so that the car goes up. But if any one of HN, N, E or LU is 1, it shall not happen. In the mathematical method that we are approaching one say then that the HN, N, E and LU must be "none" 1. And it is marked with a dash over the variable, like this:

$\overline{HN}$, $\overline{N}$, $\overline{Ö}$, $\overline{LU}$

The continuing path to the control system goes via math with a few simple rules (called Boolean algebra after George Boole, who invented this type of mathematics):

1. 1 x 1= 1, the same is true no matter how many ones that are involved, so 1 x 1 x 1 x 1 x 1 x 1 x 1 x 1= 1.

2. $\overline{0}$= 1, $\overline{1}$= 0.
3. 1 x 0= 0, even if there are a hundred ones and only one zero.
4. However, if there is a plus between two input signals, it is enough that one of them is 1, for the output to be 1, but it is never more than one (the elevator motor can not spin faster). I.e.1 + 0= 1 and 1+1+1= 1.

"The formula" that describes MU function in row 1 is:

MU= $\overline{HN}$ x DU x DN x $\overline{N}$ x $\overline{Ö}$ x $\overline{LU}$ x VU

"The formula" for MU function in row 8:

MU= HU x DU x DN x $\overline{N}$ x $\overline{Ö}$ x $\overline{LU}$

We wants the motor to spin in both cases, and therefore we can merge the two "formulas" so that it starts if any of these conditions occurs:

$$MU = \overline{HN} \times DU \times DN \times \overline{N} \times \overline{\text{Ö}} \times \overline{LU} \times VU + HU \times DU \times DN \times \overline{N} \times \overline{\text{Ö}} \times \overline{LU}$$

The formula can be written shorter by lifting all those parameters that are the same before and after the plus sign:

$$MU = DU \times DN \times \overline{N} \times \overline{\text{Ö}} \times \overline{LU} \times (\overline{HN} \times VU + HU)$$

The formula for the MN function in row 3:

$$MN = HN \times DU \times DN \times \overline{N} \times \overline{\text{Ö}} \times \overline{LN}$$

And for the MN function in row 6:

$$MN = \overline{HU} \times DU \times DN \times \overline{N} \times \overline{\text{Ö}} \times \overline{LN} \times VN$$

Merged together it becomes:

$$MN = HN \times DU \times DN \times \overline{N} \times \overline{\text{Ö}} \times \overline{LN} + \overline{HU} \times DU \times DN \times \overline{N} \times \overline{\text{Ö}} \times \overline{LN} \times VN$$

Shortened:

$$MN = DU \times DN \times \overline{N} \times \overline{\text{Ö}} \times \overline{LN} \times (HN + \overline{HU} \times VN)$$

Row 4 and 5 in table 4 describes that when there is an overload or if the emergency stop is pressed, the motor shall not spin, but we have already included that above, since as a prerequisite for MU or MN = 1 is that N and Ö are "not" 1.

Row 2 describes that the upper electric door lock (EU) should be deactivated only when the upper location contact (LU) is activated. It is described by:

$$EU = \overline{LU}$$

And corresponding for the lower door lock (row 7):

$$EN = \overline{LN}$$

The final output signal, i.e. the indicator lamp for overload (LÖ), shall be activated only if the wave (Ö) is activated:

LÖ= Ö

In summary:

$$MU = DU \times DN \times \overline{N} \times \overline{Ö} \times \overline{LU} \times (\overline{HN} \times VU + HU)$$

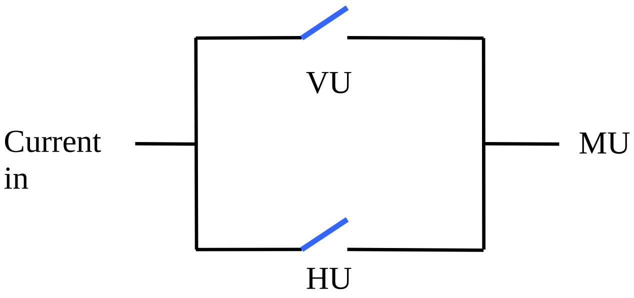
$$MN = DU \times DN \times \overline{N} \times \overline{Ö} \times \overline{LN} \times (HN + \overline{HU} \times VN)$$

$$EU = \overline{LU}$$

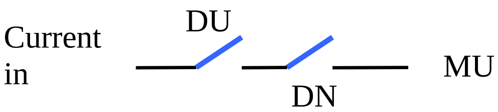
$$EN = \overline{LN}$$

$$LÖ = Ö$$

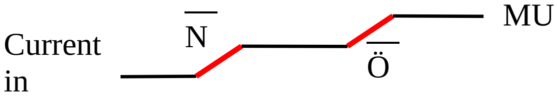
This may seem pretty silly, but the good thing is that the function can then be built with simple contacts[3] linked in a combination corresponding to the previously described mathematical rules.




With these two contacts the function: the elevator motor gets power if either one or both of the contacts HU and VU are closed (MU = HU + VU).




If the contacts connected like this: the motor gets power only if both of the contacts DU and DN are closed
(MU= DU x DN).

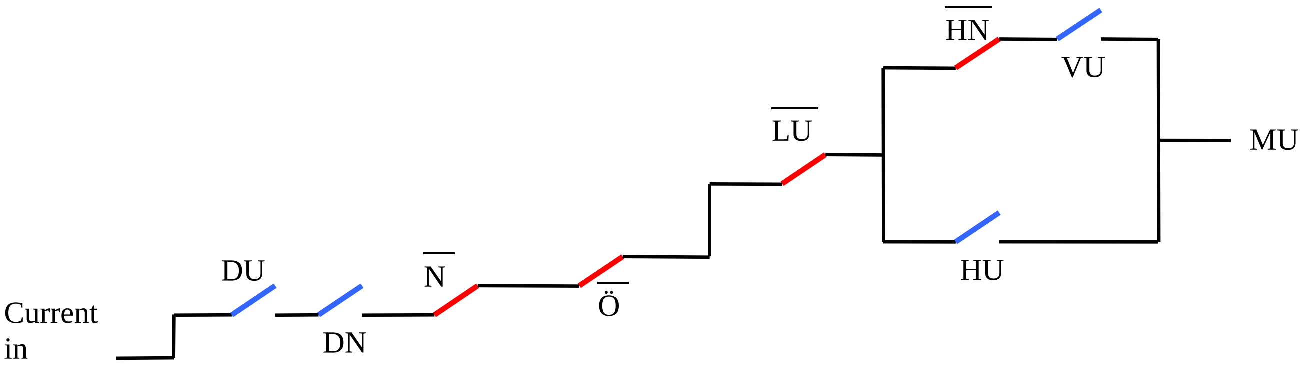



This coupling powers the motor only when both contacts are open

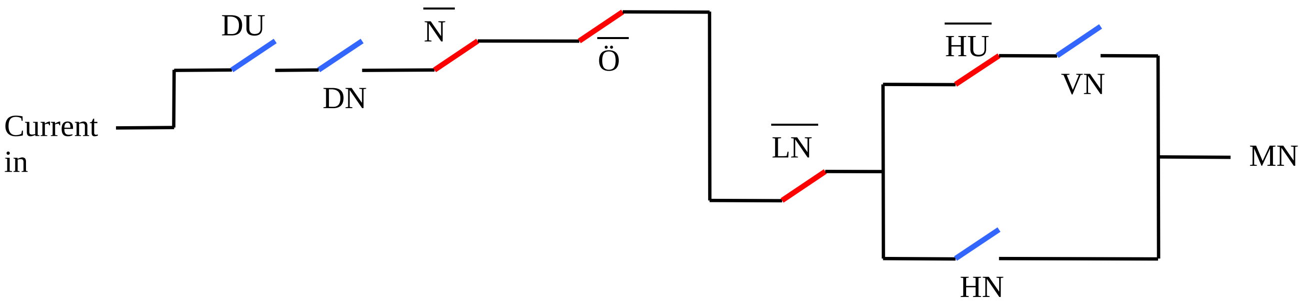
(MU= $\overline{N} \times \overline{Ö}$).

3. In real elevators the power to the engine does not pass through door contacts or other actuators because, among other things it would take so thick cables since the engine requires a lot of power. Moreover, the elevator is less dangerous if there is less voltage in the contacts (usually 24 V). Normally one build a 24 V network in the elevator, which in turn drives relays that break and release the power to the elevator motor.

$MU = DU \text{ x } DN \text{ x } \overline{N} \text{ x } \overline{\ddot{O}} \text{ x } \overline{LU} \text{ x } (\overline{HN} \text{ x } VU + HU)$:



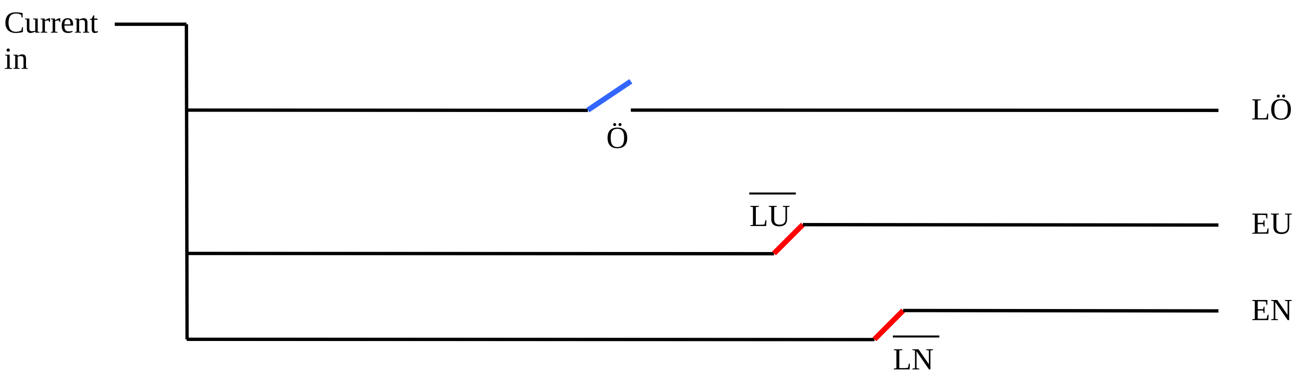

$MN = DU \text{ x } DN \text{ x } \overline{N} \text{ x } \overline{\ddot{O}} \text{ x } \overline{LN} \text{ x } (HN + \overline{HU} \text{ x } VN)$:



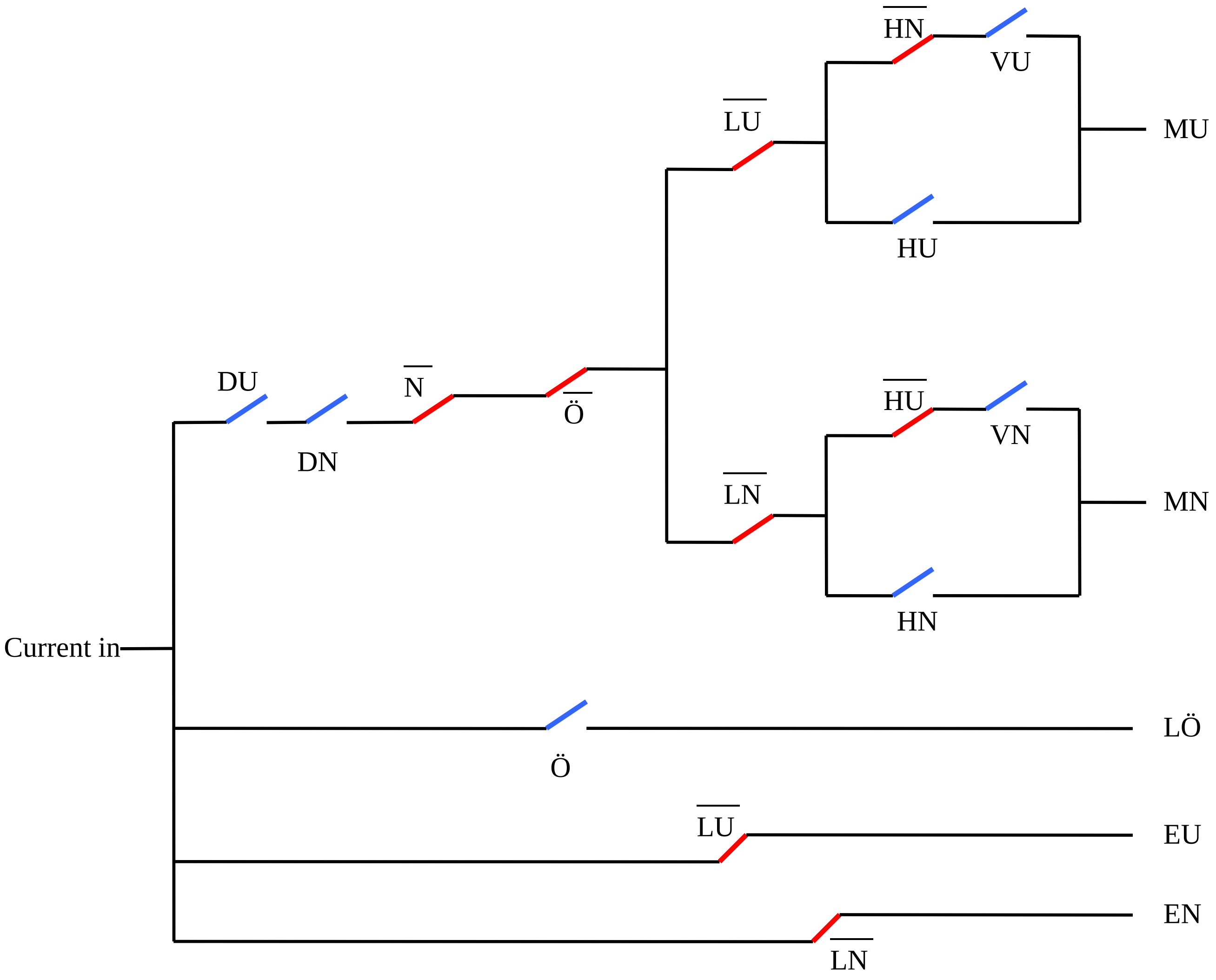

$EU = \overline{LU}$

$EN = \overline{LN}$

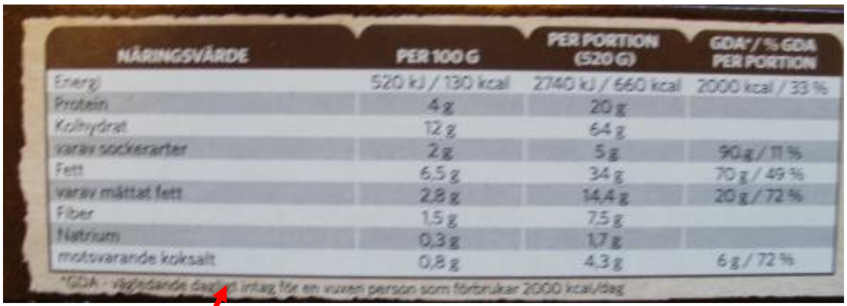
$L\ddot{O} = \ddot{O}$



And the whole control system:



We also consume energy, provided to us through the food, in the same way that gasoline contains the energy for a spark ignition engine. But we measure our energy in kilojoules (kJ) instead of kWh.



| NÄRINGSVÄRDE | PER 100 G | PER PORTION (520 G) | GDA*/ % GDA PER PORTION |
|---|---|---|---|
| Energi | 520 kJ / 130 kcal | 2740 kJ / 660 kcal | 2000 kcal / 33 % |
| Protein | 4 g | 20 g | |
| Kolhydrat | 12 g | 64 g | |
| varav sockerarter | 2 g | 5 g | 90 g / 11 % |
| Fett | 6,5 g | 34 g | 70 g / 49 % |
| varav mättat fett | 2,8 g | 14,4 g | 20 g / 72 % |
| Fiber | 1,5 g | 7,5 g | |
| Natrium | 0,3 g | 1,7 g | |
| motsvarande koksalt | 0,8 g | 4,3 g | 6 g / 72 % |

*GDA - vägledande dagligt intag för en vuxen person som förbrukar 2000 kcal/dag


One serving of taco gratin contains 2740 kJ of energy, which is a little more than half a kWh.

4 000 kJ is about 1 kWh

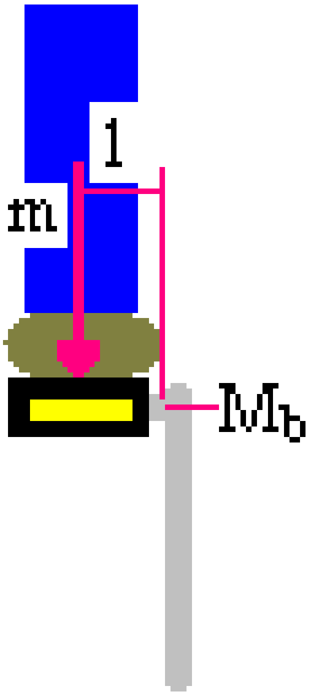
A man who lies still and weighs 70 kg burns about 5 kJ/min, which primarily is converted to heat the body. For him, it then takes about:

$$t = \frac{2.740 \text{ kJ}}{5 \text{ kJ/min}} = 548 \text{ minutes}$$

which is 548 min/60 min/h= about 9 h to consume the energy (except that the digestion itself increases the consumption)

If he instead walked with a speed of 5 km/h, his consumption would rise to 17 kJ/min and the gratin would then be burned in 161 minutes. He would get rid of the energy even faster if he ran at a speed of 10 km/h, since it would give a burning of perhaps 45 kJ/min.

Some are forced to sometimes make strength calculations, though it is usually nothing to get nervous about. Anyone who, for example, constructs a bicycle pedal should check that its weakest point stand up to even a heavy rider.
The weakest point is somewhere in the cycle where the load (called the bending torque, abbreviated Mb) is high, while the design is weak. Which is probably where the pedal is attached to the crankshaft. Therefore, the designer calculates the bending torque at that point when, for example, a person with a weight of 100 kg stands with all his weight on the pedal:




$M_b = m \times g \times l$

$M_b$= bending torque
m= the persons weight= 100 kg
g= weight acceleration= 9.81 $m/s^2$
l= the distance from the mid point of the load= 5 cm= 0.05 m

$M_b$= 100 x 9.81 x 0.05= 49 Nm= 49 000 Nmm

The bending torque creates a tension in the pedal attachment, called bending stress (abbreviated $o_b$) and must not exceed the tension that the material can withstand. How large the bending stress becomes depends on how the attachment is designed, i.e. how big is the designs bending resistance.

$o_b = \frac{M_b}{W}$ where:
$o_b$= bending stress, given in $N/mm^2$

W= bending resistance, which for a cylindrical component is:

$$W = \frac{\pi \times d^3}{32}$$

Say that the pin has a diameter of 10 mm at the narrowest point:

$W = \frac{\pi \times 10^3}{32} = 98\ mm^3$, thus the bending stress becomes:

$$o_b = \frac{49\ 000}{98} = 500\ N/mm^2$$

Which is quite a lot, so the manufacturer of the bike has to select a strong material, since if looking in a table with strength data for steel, one would find that some types of steel can not withstand that kind of bending stress. And then we have not even considered that there are people who weigh well over 100 kg.

Anyone who deals with accounting a lot would, beyond addition, subtraction, multiplication and division, also need to know some about percentages. Because VAT it includes in most purchases made, and that cost shall be deducted from the tax that most business owners add to the price of whatever they sell. When its time to pay the access VAT to the government, they add together all the VAT they have paid on their purchases and reports the total (called ingoing VAT). Additionally they sums up all the VAT that they in their turn has received from their customers (called outgoing VAT). Finally, do the following calculation:

VAT to be paid to the government = outgoing VAT - ingoing VAT

For most companies, the sum is greater than zero because they sell more than they buy, and thus there will be money for the treasury. But if the contrary occur they will instead get money back from the government.

As if to complicate things a little different goods and services are subject to different VAT. On bank services, insurances, healthcare and some rents there is no VAT at all. On food there is 12% VAT, while on most of the rest it is 25% VAT.
On most receipts of today it booth the VAT and the price without VAT (called moms in Swedish) is written and thus it’s no problem.


Systembolaget
Folkungagatan 56
S-116 22 STOCKHOLM
08/640 20 90
OrgNr: 556059-9473
Förs : 0165anhj But: 0165 Nr: 8154
Datum : 2012-01-20 14:40 Ka: 1

High Commissioner 40% 700 ml
20444-01 203,00
Lazy Lizard Merlot Cab papp 1 L
2256-01 63,00
Rosie Rosé papp 1l
6661-01 59,00

TOTAL 325,00
Totalt antal artiklar: 3

| Moms% | Moms | Netto | Brutto |
|---|---|---|---|
| 25,00 | 65,00 | 260,00 | 325,00 |
| Summa | 65,00 | 260,00 | 325,00 |

Mottaget Kontant 500,00
Åter Kontant 175,00

Två veckors öppet köp mot
uppvisande av kvitto.

But in cases where there is only a grand total including VAT it is a little "tricky". But the trick is simple:

Paid VAT (25%) = grand total x 0.2, which on my purchase at the liquor store is:

Paid VAT (25%) = 325 x 0.2 = 65 SEK.

The price without VAT is then, of course = grand total x 0.8 = 325 x 0.8 = 260 SEK.

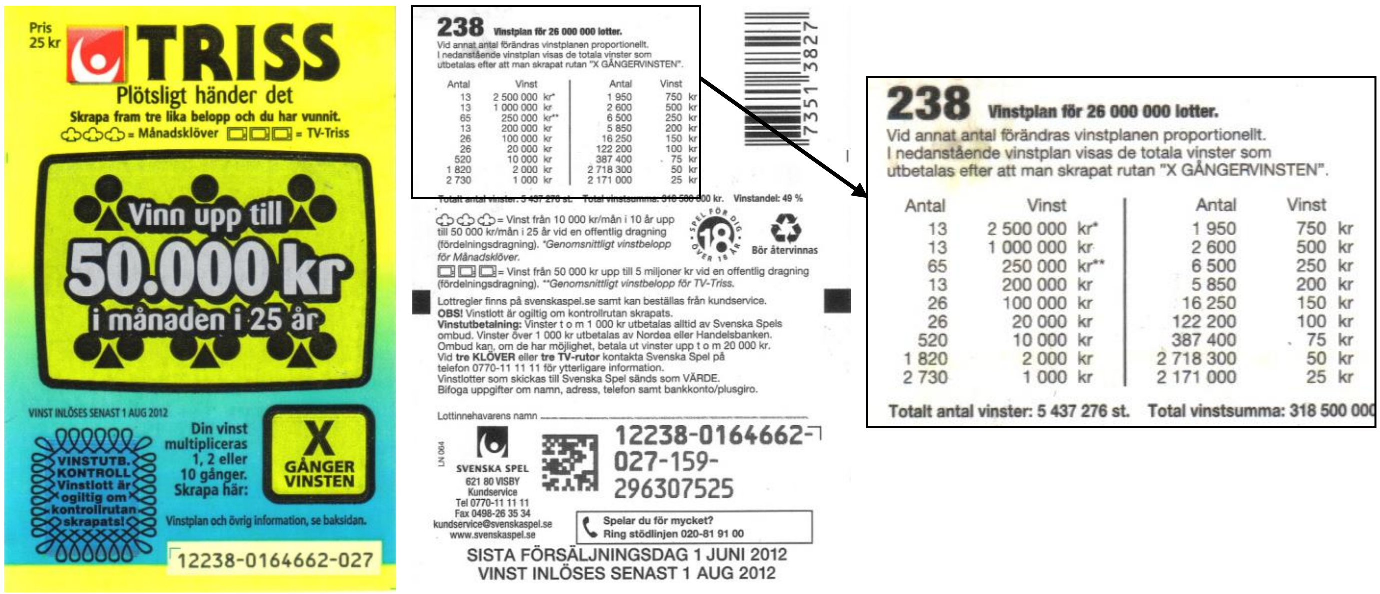
Another common case of percentage calculation, which those who are dealing with accounts may come in contact with is that they have to calculate interest on late paid invoices. The problem is that if, as in the example below, is the interest is 10% it does not mean that the customer has to pay 10% extra just because he is a few days late, instead it usually means that if the customer slips on the bill for a year it costs 10%. If he "only" pay a month late, he or she shall pay one-twelfth of 10%. There are of course usually not as much money, so it has been "solved" by adding an extra flat fee (a so-called reminder fee).

The most complicated mathematics one may meet is, in my experience, probability calculations and statistical calculations. There are an awful lot of calculation methods for things that no one other than those who chose to get involved with this comes in contact with. Though to some extent, we all do some probability calculation every now and then. Since probably everybody has drawn lots to decide about thing like, who has to take out the garbage, and then realized that the risk to be the one that has to do it, decreases with the more people that are involved in the lottery.

The same simple math applies to regular lotteries (provided that they are not rigged).

$$\text{Chances of winning} = \frac{\text{number of wins}}{\text{number of tickets}}$$

A simple division that is! Though the results can be difficult to understand if there are many lottery tickets.




According to the winning plan on the back of this lottery ticket there are 5 437 276 winning tickets and 26 million tickets. Which means that:

Chance to win = 5 437 276/26 000 000 = 0.21, which is about 1/5.

The chance to win on this lottery ticket is thus rather high for those who purchase a ticket. Though unfortunately 4 839 300 of the winnings are on only 50 SEK or less.

But someone who asks a few class mates about which British football team like the most, and then wants to draw a conclusion about which team that is the most popular in the class, may all of a sudden have passed the border into very advanced mathematical calculations. If he or she asks the whole class and all everybody answer, the questioner does not need make any statistical calculations at all. But if only, say, half the class responds, it forces him/her to make a guess about to which teams the rest might prefer.

Let's say there are 20 students in the class, and 10 responded. Of these three prefer Arsenal, four Manchester United, two Chelsea and one do not care at all. A simple guess is that Manchester United is the most popular tem, and it is true among the 10 respondents. Though it maybe was because all the Manchester United fans happened to be there when the question was asked, the rest of the class is perhaps Tottenham fans are or why not Nottingham Forest. If the author can not ask those present about what the absent ones prefer, or wait until they comes back, he can use statistical methods to calculate how big the chance is that Manchester United really is the most popular team, but then it suddenly becomes quite tricky.

Chefs also need to be able to count. First and foremost, they must keep count of how much of something that they have poured into the pot, which is easy for anyone who can add. In addition, they must be able to convert amounts in the recipe to suit the number of guests. The latter requires multiplication or division, depending on the number of guests. It is not that hard as long as it is all about, for example, three teaspoon cardamom instead of one.

Lamb Balls with coriander for 4 people

400 grams of minced meat preferably lamb
½ -1 teaspoon of salt
1 teaspoon cumin (preferably fresh grounded)
1 teaspoon coriander (preferably fresh grounded)
½ dl of chopped parsley
3 tablespoons plain yogurt

For the sauce
3 tablespoons oil
1 cinnamon stick
1 teaspoon cardamom
1 large yellow onion
1 teaspoon ginger (or rather a piece of fresh ditto)
1 teaspoon coriander (preferably fresh grounded)
1 teaspoon cayenne
1 teaspoon cumin (preferably fresh grounded)
3 tablespoons tomato paste
4-6 cloves of garlic
3 tablespoons plain yogurt
2 dl water
½ teaspoon of salt

Rice according to taste and quantity according to the guest’s appetite.

Though chefs will also be forced to deal with more advanced mathematics that involves rather complicated estimates.

400 grams of minced meat
½ -1 teaspoon salt *How many teaspoons are there in a decilitre?*
1 teaspoon of cumin
1 teaspoon coriander
½ dl of chopped parsley
3 tablespoons plain yogurt
For the sauce
3 tablespoons of oil *How many grams of solid margarine is equivalent to 3 tablespoons of oil?*
1 cinnamon stick *How many teaspoons grounded cinnamon corresponds to a cinnamon stick?*
1 teaspoon cardamom
1 large yellow onion *How many small yellow onions corresponds to a big one?*
1 teaspoon ginger
1 teaspoon coriander
1 teaspoon cayenne
1 teaspoon of cumin
3 tablespoons tomato paste
4-6 cloves of garlic
3 tablespoons plain yogurt *How much ordinary cooking spoons is that?*
2 dl water
½ teaspoon of salt

Rice at will and quantity *How many decilitre rice will each guest eat?*